The appearance of a jump (a shock wave) in the vacuum is characteristic for loading processes in materials undergoing phase transitions [3]. In the case of graphite this sudden change indicates simultaneous graphitization of diamond into which the graphite is converted during the loading process.

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STABILITY OF A THIN ELECTRIC ARC

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UDC 533.951.8:621.301.431

1. Introduction. Assuming a thermal plasma and neglecting the emission and density variation due to electromagnetic forces, we write the dimensionless equations for a steady electric arc burning in a cylindrical channel as follows [1]:

$$\begin{gathered} r^{-1}[r\lambda(T)T']' + E^2\sigma(T) = 0, \\ r^{-1}(rH)' = \sigma(T),\ \rho T = 1,\ c_p = c_p(T),\ \mu = \mu(T), \\ E = \text{const},\ H_\varphi = EH. \end{gathered} \tag{1.1}$$

One can select

$$T|_{r=0} = 1,\ T'|_{r=0} = H|_{r=0} = 0,\ T|_{r=1} = T_R \tag{1.2}$$

as the boundary conditions. The constant E is determined from the three boundary conditions for the first equation of the system (1.1). Here T, ρ, σ, E, H_φ, λ, c_p, and μ are the dimensionless temperature, density, electrical conductivity, electric field intensity applied along the z axis, φ-th component of the intrinsic magnetic field intensity, thermal conductivity coefficient, specific heat at constant pressure, and the dynamic viscosity coefficient; T_R is the dimensionless temperature on the channel wall; and r, φ, z are cylindrical coordinates; here and below a prime denotes a derivative with respect to r.

The values of the corresponding parameters on the channel axis (with subscript m) are the scale factors of T, ρ, σ, λ, c_p, and μ. The scale factors of the electric field intensity and the magnetic field intensity are

$$E_m = \sqrt{\lambda_m T_m/\sigma_m}/R_m,\ H_{\varphi m} = E_m\sigma_m R_m.$$

The stability of an electric arc has been investigated in [1] with respect to symmetrical perturbations with viscosity taken into account. It turned out that for the critical curves (we will mark the critical parameters below with a subscript c) which separate the stable regions from the unstable ones the phase velocity of the perturbations is equal to zero, and the stability boundary is determined by the value of the product of the Stewart number by the viscosity parameter. The equations along with the boundary conditions are of the form

$$\begin{gathered} p' = -E^2Q[H(\sigma e + (d\sigma/dT)\theta/\lambda) + \sigma h] + 2r^{-1}(r\mu v')' - \\ -\mu(2/r^2 + k^2)v + \mu w' - (2/3)\{\mu[r^{-1}(rv)' + w]'\}', \\ r^{-1}(r\mu w')' = -k^2p - E^2k^2QHh + k^2r^{-1}(r\mu v)' + \\ + (4/3)k^2\mu w - (2/3)k^2\mu r^{-1}(rv)', \\ r^{-1}(r\rho v)' = -\rho w, \\ r^{-1}(r\theta')' = c_pT'\rho v + k^2\theta - E^2(2\sigma e + (d\sigma/dT)\theta/\lambda), \\ r^{-1}(rh)' = \sigma e + (d\sigma/dT)\theta/\lambda,\ e' = k^2h/\sigma; \end{gathered} \tag{1.3}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 108-118, March-April, 1979. Original article submitted March 10, 1978.

$$v = w' = \theta' = h = 0 \text{ at } r = 0,$$
$$v = w = \theta = h = 0 \text{ at } r = 1 \qquad (1.4)$$

in this case. Here θ/λ, v, w/ik, Pp, Eh, Ee are the amplitudes of the perturbations, respectively, of the temperature, radial velocity component, axial velocity component, pressure, φ-th component of the magnetic field intensity, and the z-component of the electric field intensity; k is the wave number, Q = SP is a parameter which characterizes the stability boundary; $S = \mu_e\sigma_m\rho_m T_m c_{pm}^2 R_m^2/\lambda_m$ is the Stewart number; $P = \rho_m V_m R_m/\mu_m = \lambda_m/\mu_m c_{pm}$ is the viscosity parameter; and $\mu_e = 4\pi \cdot 10^{-7}$ kg · m/kl is the magnetic permeability. The scale factors of the time, velocity, and pressure are

$$t_m = \rho_m c_{pm} R_m^2/\lambda_m, \; V_m = R_m/t_m, \; p_m = \rho_m V_m^2,$$

respectively. The boundary conditions (1.4) are imposed from considerations of the boundedness of the functions at r = 0 and of the fact that the channel boundary is an impermeable non-current-conducting surface at constant temperature. One should note that it is possible to lower the order of the system (1.3). For example, it is possible to determine v' from the third equation of the system (1.3) and to substitute it into the first equation. This fact should be kept in mind in what follows.

The problem (1.1)-(1.4) will be investigated in this paper in the case in which the channel is divided into electrically conducting and nonconducting zones and the radius of the electrically conducting zone (the arc radius is r_σ) is small ($r_\sigma \ll 1$):

$$\sigma(T) = \begin{cases} \sigma(T) & \text{for } T > T_\sigma, \\ 0 & \text{for } T < T_\sigma \end{cases} \qquad (1.5)$$

(T_σ is the value of the temperature below which the electrical conductivity is equal to zero). It is evident that the solution for the perturbed magnetic field outside the electric arc is

$$h = 0 \text{ for } r > r_\sigma. \qquad (1.6)$$

Specifying the electrical conductivity in the form of a continuous function of the temperature (and so, of the radius), we obtain that the perturbed magnetic field intensity is equal to zero,

$$h|_{r=r_{\sigma 0}} = 0, \qquad (1.7)$$

on the unperturbed surface of the electrically conducting zone ($r = r_{\sigma 0}$).

With the use of Eqs. (1.6) and (1.5) we write the equations for the electrically conducting zone in the form

$$\begin{aligned} p' &= 2r^{-1}(r\mu v')' - \mu(2/r^2 + k^2)v + \mu w' - \\ &-(2/3)\{\mu[r^{-1}(rv)' + w]\}', \\ r^{-1}(r\mu w')' &= -k^2 p + k^2 r^{-1}(r\mu v)' + \\ &+ (4/3)k^2\mu w - (2/3)k^2\mu r^{-1}(rv)', \\ r^{-1}(r\rho v') &= -\rho w, \; r^{-1}(r\theta')' = c_p T'\rho v + k^2\theta. \end{aligned} \qquad (1.8)$$

The solutions for the perturbations obtained in the electrically conducting and nonconducting zones should be spliced on the unperturbed surface of the electric arc ($r = r_{\sigma 0}$). Selecting Q in an appropriate way, it is possible to obtain a nontrivial solution of the problem (1.3), (1.8), (1.4), and (1.7); i.e., in this case Q fulfills the role of an eigenvalue.

2. Solution of the Problem for $|\ln r_{\sigma 0}|^{-1} \ll T_R$. Usually, the temperature of the channel wall in electric arcs is significantly less than the temperature on the axis; i.e., $T_R \ll 1$. The condition

$$|\ln r_{\sigma 0}|^{-1} \ll T_R \qquad (2.1)$$

denotes the limiting case of as thin an arc as desired; it is assumed in the investigation of the burning stability of such an arc that the remaining parameters are fixed. The solution of the problem (1.3), (1.8), (1.4), and (1.7) under the condition (2.1) is of independent interest and also very significant from the methodological point of view.

For simplicity's sake the stability investigation will be conducted with

$$\mu = c_p = \lambda = 1. \tag{2.2}$$

We then note that the unperturbed temperature for the electrically conducting zone is determined by the formula

$$T = r_{\sigma 0} T' |_{r=r_{\sigma 0}} \ln r + T_R. \tag{2.3}$$

We introduce the notation

$$\varepsilon^2 = 1 - T_\sigma,\ \varepsilon_1 = \varepsilon / E, \tag{2.4}$$

and also the new functions

$$\tilde{T} = (T - T_\sigma)/\varepsilon^2,\ \tilde{H} = H/\varepsilon_1. \tag{2.5}$$

In the limiting process $\varepsilon_1 \to 0$ ($r_{\sigma 0} \to 0$), ζ is fixed, and the inner coordinate (the terminology is taken from [2]) has the form

$$\zeta = r/\varepsilon_1. \tag{2.6}$$

Then the equations for the unperturbed parameter of the electric arc (1.1) with the boundary conditions (1.2) are rewritten in the form

$$\zeta^{-1}(d/d\zeta)(\zeta d\tilde{T}/d\zeta) + \sigma(\tilde{T}) = 0,\ \zeta^{-1}(d/d\zeta)(\zeta\tilde{H}) = \sigma(\tilde{T}); \tag{2.7}$$

$$d\tilde{T}/d\zeta|_{\zeta=0} = \tilde{H}|_{\zeta=0} = 0,\ \tilde{T}|_{\zeta=0} = 1 \tag{2.8}$$

and the electrical conductivity is written in the form

$$\sigma = \begin{cases} \sigma(\tilde{T}) & \text{for} \quad \tilde{T} > 0, \\ 0 & \text{for} \quad \tilde{T} < 0. \end{cases} \tag{2.9}$$

A boundary condition equivalent to $T|_{r=1} = T_R$ is omitted in (2.8), since the solution written in the variable ζ is interesting only for $r \ll 1$. The value of ζ at which $\tilde{T}$ becomes equal to zero is denoted as γ; i.e.,

$$r_{\sigma 0}/\varepsilon_1 = \gamma. \tag{2.10}$$

It is evident from the form of the problem (2.7) and (2.8) that γ, and also $\gamma d\tilde{T}/d\zeta|_{\zeta=\gamma}$, do not depend on $r_{\sigma 0}$. One can obtain from Eq. (2.3) with the help of Eqs. (2.4)-(2.6) and (2.10) a relation between ε^2 and the arc radius

$$\varepsilon^2 = (1 - T_R)/(\gamma d\tilde{T}/d\zeta|_{\zeta=\gamma} \ln r_{\sigma 0} + 1), \tag{2.11}$$

whence it is clear that $\varepsilon^2 \sim |\ln r_{\sigma 0}|^{-1}$ as $r_{\sigma 0} \to 0$.

Let us rewrite the system of equations (1.3), using the new variable ζ and Eqs. (2.2), (2.4), and (2.5):

$$\begin{gathered} dp/d\zeta = -\varepsilon_1 E^2 Q[\varepsilon_1 \tilde{H}(\sigma e + (d\sigma/d\tilde{T})\theta/\varepsilon^2) + \sigma h] + \\ + \varepsilon^{-1}(4/3)(d/d\zeta)[\zeta^{-1}(d/d\zeta)(\zeta v)] - \varepsilon_1 k^2 v + (1/3) dw/d\zeta, \\ \zeta^{-1}(d/d\zeta)(\zeta dw/d\zeta) = \varepsilon_1^2 (4/3) k^2 w + \varepsilon_1 (1/3) k^2 \zeta^{-1} (d/d\zeta)(\zeta v) - \\ - \varepsilon_1^2 k^2 p - \varepsilon_1^3 E^2 Q k^2 \tilde{H} h, \\ \zeta^{-1}(d/d\zeta)\{\zeta v/[1 - \varepsilon^2(1 - \tilde{T})]\} = -\varepsilon_1 w/[1 - \varepsilon^2(1 - \tilde{T})], \\ \zeta^{-1}(d/d\zeta)(\zeta d\theta/d\zeta) = \varepsilon^2 \varepsilon_1 (d\tilde{T}/d\zeta) v/[1 - \\ - \varepsilon^2(1 - \tilde{T})] + \varepsilon_1^2 k^2 \theta - \varepsilon^2 (2\sigma e + (d\sigma/d\tilde{T})\theta/\varepsilon^2), \\ \zeta^{-1}(d/d\zeta)(\zeta h) = \varepsilon_1(\sigma e + (d\sigma/d\tilde{T})\theta/\varepsilon^2), \\ de/d\zeta = \varepsilon_1 k^2 h/\sigma. \end{gathered} \tag{2.12}$$

The ideas of perturbation methods (e.g., see [2]) are used in connection with the solution of the problem (1.3), (1.8), (1.4), and (1.7) as $r_{\sigma 0} \to 0$. It is possible to construct an asymptotic expansion of the eigenfunctions and eigenvalues which is valid at the boundary $r = 0$ (interior expansion) as follows:

$$\begin{gathered} p = (2T_R Q_0/\beta^2)\, \varepsilon_1^{-2} \ln \varepsilon_1 [\tilde{p}_0(\zeta) + \nu_{p1}(\varepsilon_1)\tilde{p}_1(\zeta) + \nu_{p2}(\varepsilon_1)\tilde{p}_2(\zeta) + \ldots], \\ w = (2T_R Q_0/\beta^2)\, k^2 \ln \varepsilon_1 \left[\tilde{w}_0(\zeta) + \frac{1}{\ln \varepsilon_1}\tilde{w}_1(\zeta) + \nu_{w2}(\varepsilon_1)\tilde{w}_2(\zeta) + \ldots\right], \end{gathered} \tag{2.13}$$

$$v = (2T_R Q_0/\beta^2)\, k^2 \varepsilon_1 \ln^2 \varepsilon_1 \left[\widetilde{v}_0(\zeta) + \frac{1}{\ln \varepsilon_1}\widetilde{v}_1(\zeta) + \nu_{v2}(\varepsilon_1)\widetilde{v}_2(\zeta) + \ldots\right],$$
$$\theta = \varepsilon^2 [\widetilde{\theta}_0(\zeta) + \nu_{\theta 1}(\varepsilon_1)\widetilde{\theta}_1(\zeta) + \ldots],\ h = \varepsilon_1 [\widetilde{h}_0(\zeta) + \nu_{h1}(\varepsilon_1)\widetilde{h}_1(\zeta) + \ldots],\ e = \widetilde{e}_0(\zeta) + \nu_{e1}(\varepsilon_1)\widetilde{e}_1(\zeta) + \ldots,$$
$$Q = (2T_R/\beta^2)\, \varepsilon_1^{-2} \varepsilon^{-2} \ln \varepsilon_1 [Q_0 + \nu_{Q1}(\varepsilon_1) Q_1 + \ldots], \tag{2.13}$$

where $\nu_{p,n+1}/\nu_{pn} \to 0$, $\nu_{w,n+1}/\nu_{wn} \to 0, \ldots$ as $\varepsilon_1 \to 0$, and

$$\beta^2 = -\gamma d\widetilde{T}/d\zeta|_{\zeta=\gamma}. \tag{2.14}$$

Substituting the asymptotic expansions (2.13) into the system of equations (2.12), we obtain

$$\begin{gathered}
d\widetilde{p}_0/d\zeta = -\widetilde{H}(\sigma\widetilde{e}_0 + (d\sigma/d\widetilde{T})\widetilde{\theta}_0) - \sigma\widetilde{h}_0,\\
\zeta^{-1}(d/d\zeta)(\zeta d\widetilde{w}_1/d\zeta) = -\widetilde{p}_0 - \widetilde{H}_0\widetilde{h}_0,\\
\zeta^{-1}(d/d\zeta)(\zeta d\widetilde{\theta}_0/d\zeta) = -2\sigma\widetilde{e}_0 - (d\sigma/d\widetilde{T})\widetilde{\theta}_0,\\
\zeta^{-1}(d/d\zeta)(\zeta\widetilde{h}_0) = \sigma\widetilde{e}_0 + (d\sigma/d\widetilde{T})\widetilde{\theta}_0,\\
d\widetilde{e}_0/d\zeta = 0,\ \zeta^{-1}(d/d\zeta)(\zeta d\widetilde{w}_0/d\zeta) = 0,\ \zeta^{-1}(d/d\zeta)(\zeta\widetilde{v}_0) = -\widetilde{w}_0,\\
\zeta^{-1}(d/d\zeta)(\zeta\widetilde{v}_1) = -\widetilde{w}_1 - ((1 - T_R)/\beta^2)(d\widetilde{T}/d\zeta)\widetilde{v}_0.
\end{gathered} \tag{2.15}$$

Outside the conductivity region ($\zeta > \gamma$) the solutions of the system of equations (2.15) will be determined by the formulas

$$\begin{gathered}
\widetilde{p}_0 = C_1,\ d\widetilde{w}_1/d\zeta = C_1\zeta/2 + C_2/\zeta,\\
\widetilde{w}_1 = C_1\zeta^2/4 + C_2 \ln\zeta + C_3,\ d\widetilde{\theta}_0/d\zeta = C_5/\zeta,\\
\widetilde{\theta}_0 = C_5 \ln\zeta + C_6,\ \widetilde{w}_0 = C_7,\ \widetilde{v}_0 = -C_7\zeta/2,\\
\widetilde{v}_1 = -C_1\zeta^3/16 - C_2((\zeta/2)\ln\zeta - \zeta/4) -\\
-C_3\zeta/2 + C_4/\zeta - C_7(1 - T_R)\zeta/4,
\end{gathered} \tag{2.16}$$

where $C_1, C_2, \ldots, C_7$ are constants determined from the splicing conditions of the solutions at $\zeta = \gamma$; e.g.,

$$\begin{gathered}
C_1 = \widetilde{p}_0(\gamma),\ C_2 = \gamma d\widetilde{w}_1/d\zeta|_{\zeta=\gamma} - \widetilde{p}_0(\gamma)\gamma^2/2,\\
C_3 = \widetilde{w}_1(\gamma) - \widetilde{p}_0(\gamma)\gamma^2/4 - C_2\ln\gamma, \ldots .
\end{gathered}$$

For $r > r_{\sigma 0}$ Eqs. (1.8) can be written in the form

$$\begin{gathered}
p' = [(4/3)\varepsilon^2\beta^2/r(T_R - \varepsilon^2\beta^2\ln r)]w - w' -\\
-[k^2 - (8/3)\varepsilon^2\beta^2/r^2(T_R - \varepsilon^2\beta^2\ln r)]v,\\
r^{-1}(rw')' = k^2 w - k^2 p - [(k^2/3)\varepsilon^2\beta^2/r(T_R - \varepsilon^2\beta^2\ln r)]v,\\
r^{-1}(rv)' = -w - [\varepsilon^2\beta^2/r(T_R - \varepsilon^2\beta^2\ln r)]v,\\
r^{-1}(r\theta')' = k^2\theta - [\varepsilon^2\beta^2/r(T_R - \varepsilon^2\beta^2\ln r)]v
\end{gathered} \tag{2.17}$$

with the use of Eqs. (2.2), (2.3), (2.5), and (2.14).

One can construct an asymptotic expansion of the eigenfunctions, which is valid at the boundary $r = 1$ (exterior expansion) as follows:

$$\begin{gathered}
p = (2T_R k/\beta^2)\ln\varepsilon_1[p_0(r) + \mu_{p1}(\varepsilon_1)p_1(r) +\\
+ \mu_{p2}(\varepsilon_1)p_2(r) + \ldots],\\
w = (2T_R k/\beta^2)\ln\varepsilon_1[w_0(r) + \mu_{w1}(\varepsilon_1)w_1(r) + \ldots],\\
v = (2T_R k/\beta^2)\ln\varepsilon_1[v_0(r) + \mu_{v1}(\varepsilon_1)v_1(r) + \ldots],\\
\theta = \varepsilon^2\ln\varepsilon_1[\theta_0(r) + \mu_{\theta 1}(\varepsilon_1)\theta_1(r) + \ldots],
\end{gathered} \tag{2.18}$$

where $\mu_{p,n+1}/\mu_{pn} \to 0$, $\mu_{w,n+1}/\mu_{wn} \to 0, \ldots$ as $\varepsilon_1 \to 0$.

Substituting Eqs. (2.18) into the system (2.17), we obtain the following system of equations in the zeroth approximation:

$$p_0' = -w_0' - k^2v_0,\ r^{-1}(rw_0')' = k^2w_0 - k^2p_0,$$
$$r^{-1}(rv_0)' = -w_0,\ r^{-1}(r\theta_0')' = -(2k/r)v_0 + k^2\theta_0. \quad (2.19)$$

The solution of the system (2.19) is (for example, see [3])

$$\begin{aligned}
v_0 &= B_1I_1(kr) + B_2K_1(kr) + B_3krI_0(kr) + B_4krK_0(kr),\\
w_0 &= -B_1kI_0(kr) + B_2kK_0(kr) - B_3k[2I_0(kr) + \\
&\quad + krI_1(kr)] - B_4k[2K_0(kr) - krK_1(kr)],\\
w_0' &= -B_1k^2I_1(kr) - B_2k^2K_1(kr) - B_3k^2[2I_1(kr) + \\
&\quad + krI_0(kr)] + B_4k^2[2K_1(kr) - krK_0(kr)],\\
p_0 &= 2B_3kI_0(kr) + 2B_4kK_0(kr),\\
\theta_0 &= -B_1I_0(kr)\ln kr + B_2K_0(kr)\ln kr - B_3krI_1(kr) + \\
&\quad + B_4krK_1(kr) + B_5I_0(kr) + B_6K_0(kr),\\
\theta_0' &= -B_1[kI_1(kr)\ln kr + I_0(kr)/r] + B_2[-kK_1(kr)\ln kr + K_0(kr)/r] - \\
&\quad - B_3k^2rI_0(kr) - B_4k^2rK_0(kr) + B_5kI_1(kr) - B_6kK_1(kr),
\end{aligned} \quad (2.20)$$

where I_0 and I_1 are the modified Bessel functions, K_0 and K_1 are the Macdonald functions, and $B_1, B_2, \ldots, B_6$ are arbitrary constants. Without dwelling on the construction of the solution (2.20), we note that it is possible to confirm its correctness with a check.

Let us consider for the splicing of the interior and exterior expansions the intermediate limit

$$\varepsilon_1 \to 0,\ r_\eta = r/\eta \text{ is fixed,} \quad (2.21)$$

where

$$\varepsilon_1 \ll \eta \ll 1 \quad \text{or} \quad \eta/\varepsilon_1 \to \infty,\ \eta \to 0.$$

In the limit (2.21) $r = \eta r_\eta \to 0$, $\zeta = (\eta/\varepsilon_1)r_\eta \to \infty$. In connection with the splicing of the first terms of the interior and exterior expansions, using the definition (2.21), Eqs. (2.10), 2.11), (2.14), (2.16), and (2.20), as well as the expressions for the functions I_0, I_1, K_0, K_1 in series form, we have (the functions p, w', w, v, θ', θ are spliced serially)

$$\begin{aligned}
&C_1 = 0,\ Q_0C_2 = 2B_4,\ 0 = B_2,\ 0 = B_1 + B_6,\\
&-C_5 = B_4 + B_5 + B_6\,(\ln 2 - C),\ -C_2 + C_7 = 0,\\
&C_3Q_0 = -B_1 - 2B_3 + B_4[2\ln(k/2) + 2C + 1],
\end{aligned} \quad (2.22)$$

where C = 0.577 ... is Euler's constant. Here $B_2 = 0$ is used in writing Eqs. (2.22). We obtain with the help of Eqs. (2.16)

$$\begin{aligned}
&\tilde{p}_0(\gamma) = 0,\ Q_0k\gamma d\tilde{w}_1/d\zeta|_{\zeta=\gamma} = \lim_{r\to 0}(rw_0'),\\
&0 = v_0|_{r=0},\ 0 = \theta_0'|_{r=0},\ -\gamma d\tilde{\theta}_0/d\zeta|_{\zeta=\gamma} = \theta_0|_{r=0},\\
&(\tilde{w}_0 - \gamma d\tilde{w}_1/d\zeta)|_{\zeta=\gamma} = 0,\ Q_0(\tilde{w}_1 - \gamma\ln\gamma d\tilde{w}_1/d\zeta)|_{\zeta=\gamma} = \lim_{r\to 0}(w_0/\ln r).
\end{aligned} \quad (2.23)$$

The zeroth and succeeding terms of the asymptotic expansions (2.13) and (2.18) (the interior and exterior expansions), should, as follows from (1.4) and (1.7), satisfy the boundary conditions

$$\begin{aligned}
&\tilde{v}_n = d\tilde{w}_n/d\zeta = d\tilde{\theta}_n/d\zeta = \tilde{h}_n = 0 \quad \text{at} \quad \zeta = 0,\ \tilde{h}_n = 0 \quad \text{at} \quad \zeta = \gamma,\\
&v_n = w_n = \theta_n = 0 \quad \text{at} \quad r = 1\ (n = 0, 1, 2, \ldots).
\end{aligned} \quad (2.24)$$

One should note that the first five equations of the system (2.15) are enough to determine the eigenvalue Q_0 [this follows from the conditions (2.23)]. One can express B_1, B_3, B_5, and B_4 from Eqs. (2.20), (2.22), and (2.24) in terms of

$$B_1 = b_1B_4,\ B_3 = b_3B_4,\ B_5 = b_5B_4, \quad (2.25)$$

where

$$b_1 = k^2[K_0(k)I_1(k) + I_0(k)K_1(k_1)]/\{kI_0(k) - -I_1(k)[2I_0(k) + kI_1(k)]\};$$

$$b_3 = \{I_1(k)[2K_0(k) - kK_1(k)] - kK_0(k)I_0(k)\}/\{kI_0(k) - I_1(k) \times \times [2I_0(k) + kI_1(k)]\}; \; b_5 = \{b_1[I_0(k) \ln k + K_0(k)] + + b_3 k I_1(k) - kK_1(k)\}/I_0(k).$$

When

$$\sigma(\widetilde{T}) = \begin{cases} \widetilde{T} & \text{for} \quad \widetilde{T} > 0, \\ 0 & \text{for} \quad \widetilde{T} < 0 \end{cases} \tag{2.26}$$

it is possible to solve the problems (2.7), (2.8), (2.15), and (2.24) analytically. The solutions for the unperturbed parameters in the electrically conducting zone are of the form

$$\widetilde{T} = J_0(\zeta), \; \widetilde{H} = J_1(\zeta) \tag{2.27}$$

for $\zeta < \gamma$; $\gamma = 2.405$ is the value of the first positive root of the equation $J_0(\gamma) = 0$, and J_0 and J_1 are Bessel functions. Using Eqs. (2.27) and (2.26), the solutions for the perturbations of the problem (2.15) and (2.24) can be written in the form

$$\begin{gathered} \widetilde{e}_0 = A_1, \; \widetilde{\theta}_0 = A_1[J_0(\zeta) - \zeta J_1(\zeta)], \; \widetilde{h}_0 = A_1 \zeta J_0(\zeta), \\ \widetilde{p}_0 = A_1 \left[J_0^2(\zeta) - \zeta J_0(\zeta) J_1(\zeta)\right], \\ d\widetilde{w}_1/d\zeta = -A_1 \zeta \left[J_0^2(\zeta) + J_1^2(\zeta)\right]; \end{gathered} \tag{2.28}$$

here only the solutions necessary for the determination of Q_0 are written. Substituting (2.28) into (2.22) and using (2.26), we obtain

$$\begin{gathered} -A_1 \gamma^2 J_1^2(\gamma) Q_0 = 2B_4, \\ A_1 \gamma J_1(\gamma) = B_4\{1 + b_5 - b_1(\ln 2 - C)\}. \end{gathered} \tag{2.29}$$

Equating the determinant of the system of equations (2.29) with respect to the unknowns A_1 and B_4 to zero, one can find Q_0.

The function $Q_0(k)$ is illustrated in Fig. 1; the stability region is denoted by the letter "s" and the instability region by the letter "i". It is evident that $|Q_0|$ reaches a minimum as $k \to \infty$; i.e., perturbations with large wave numbers are the most unstable.

The eigenfunctions of zeroth approximation in the electrically conducting region are plotted in Fig. 2, and those outside of it with $k = 4$ are plotted in Fig. 3. It follows from Figs. 2 and 3 as well as from the expansions (2.13) and (2.18) that the amplitude of the pressure perturbations increases most intensely with decreasing arc radius; the maximum pressure, and also the maximum longitudinal velocity component (maxima of the perturbation amplitudes are understood here), are found on the channel axis. The maximum of the temperature perturbation is found near the electrically conducting zone but outside of it. The maximum perturbation of the radial velocity is found outside the electrically conducting zone.

3. Solution of the Problem for $|\ln r_{\sigma 0}|^{-1} = O(T_R)$. In the case in which the wall temperature is a quantity of the first order of smallness with $|\ln r_{\sigma 0}|^{-1}$ (or with ε^2), it is possible to introduce the notation

$$T_R = \varepsilon^2 \beta^2 t_R. \tag{3.1}$$

Then using the interior variable ζ [see (2.6)] and also Eqs. (2.3)-(2.5), we construct near the boundary $r = 0$ an asymptotic expansion of the eigenfunctions and eigenvalues

$$\begin{gathered} p = Q_0 \varepsilon_1^{-2}[\widetilde{p}_0(\zeta) + \nu_{p1}(\varepsilon_1)\widetilde{p}_1(\zeta) + \nu_{p2}(\varepsilon_1)\widetilde{p}_2(\zeta) + \ldots], \\ w = k^2 Q_0 \ln \varepsilon_1 [\widetilde{w}_0(\zeta) + (\ln \varepsilon_1)^{-1}\widetilde{w}_1(\zeta) + \nu_{w2}(\varepsilon_1) \; \widetilde{w}_2(\zeta) + \ldots], \\ v = k^2 Q_0 \varepsilon_1 \ln \varepsilon_1 [\widetilde{v}_0(\zeta) + (\ln \varepsilon_1)^{-1}\widetilde{v}_1(\zeta) + \nu_{v2}(\varepsilon_1)\widetilde{v}_2(\zeta) + \ldots], \\ \theta = \varepsilon^2[\widetilde{\theta}_0(\zeta) + \nu_{\theta 1}(\varepsilon_1)\widetilde{\theta}_1(\zeta) + \ldots], \; h = \varepsilon_1[\widetilde{h}_0(\zeta) + \\ + \nu_{h1}(\varepsilon_1)\widetilde{h}_1(\zeta) + \ldots], \\ e = \widetilde{e}_0(\zeta) + \nu_{e1}(\varepsilon_1)\widetilde{e}_1(\zeta) + \ldots, \; Q = \varepsilon^{-2}\varepsilon_1^{-2}[Q_0 + \nu_{Q1}(\varepsilon_1) Q_1 + \ldots]. \end{gathered} \tag{3.2}$$

Substituting the asymptotic series (3.2) into Eqs. (2.12) and keeping (3.1) in mind, we obtain the system of equations (2.15), with the exception of the last equation, which has a somewhat different form and is not needed in the following.

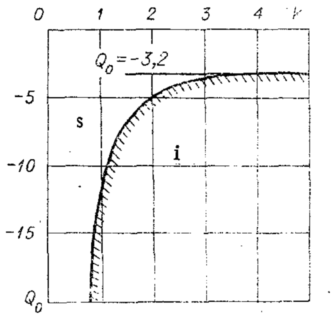

Fig. 1

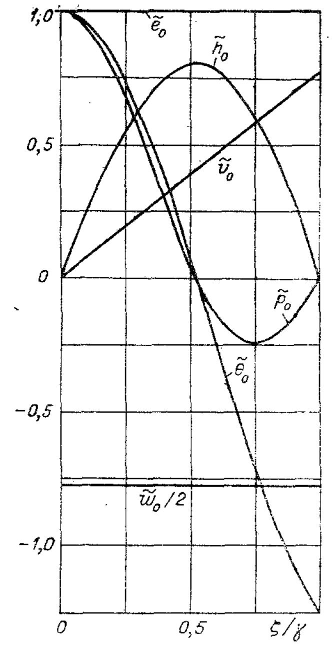

Fig. 2

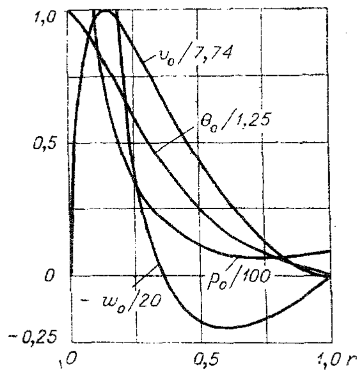

Fig. 3

An asymptotic expansion of the eigenfunctions, which is valid near the boundary $r=1$ (exterior expansion), can be represented in the form

$$p = p_0(r) + \mu_{p1}(\varepsilon_1)p_1(r) + \mu_{p2}(\varepsilon_1)p_2(r) + \ldots,$$
$$w = w_0(r) + \mu_{w1}(\varepsilon_1)w_1(r) + \ldots, \quad v = v_0(r) + \mu_{v1}(\varepsilon_1)v_1(r) + \ldots, \tag{3.3}$$
$$\theta = \theta_0(r) + \mu_{\theta 1}(\varepsilon_1)\theta_1(r) + \ldots,$$

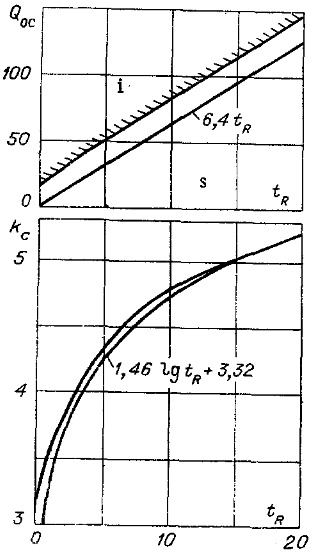

Fig. 4

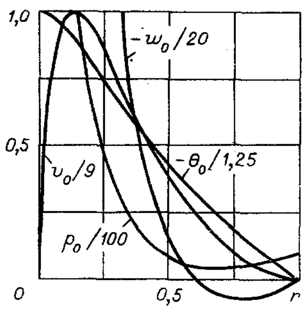

Fig. 5

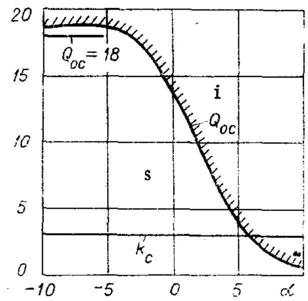

Fig. 6

where $\mu_{p,n+1}/\mu_{pn} \to 0$, $\mu_{w,n+1}/\mu_{wn} \to 0, \ldots$ as $\varepsilon_1 \to 0$. Then substituting the series (3.3) into the system (2.17), we obtain the following system of equations in the zeroth approximation:

$$
\begin{gathered}
p_0' = -w_0' + (4/3)\, w_0/r\,(t_R - \ln r) - [k^2 - 8/3r^2\,(t_R - \ln r)]\, v_0, \\
r^{-1}(rv_0)' = -w_0 - v_0/r\,(t_R - \ln r), \\
r^{-1}(rw_0')' = k^2 w_0 - k^2 p_0 - k^2 v_0/3r\,(t_R - \ln r) \\
r^{-1}(r\theta_0')' = k^2\theta_0 - v_0/r\,(t_R - \ln r).
\end{gathered} \tag{3.4}
$$

The behavior of the solutions of the system of equations (3.4) as $r \to 0$ coincides with the behavior of the solutions of the system (2.19); therefore, it is possible in connection with the splicing of the exterior and interior expansions to obtain conditions similar to (2.23):

$$
\begin{gathered}
\tilde{p}_0(\gamma) = 0, \quad k^2 Q_0 \gamma\, d\tilde{w}_1/d\zeta\,|_{\zeta=\gamma} = \lim_{r\to 0}(rw_0'), \\
0 = v_0\,|_{r=0}, \quad 0 = \theta_0'\,|_{r=0}, \quad \gamma\, d\tilde{\theta}_0/d\zeta\,|_{\zeta=\gamma} = \theta_0\,|_{r=0}
\end{gathered} \tag{3.5}
$$

(only the splicing conditions necessary for the determination of the eigenvalue Q_0 are written out here). It is also evident that the zeroth and subsequent terms of the interior and exterior expansions (3.2) and (3.3) should satisfy the boundary conditions (2.24). The systems of Equations (2.15) and (3.4) along with the conditions (2.24) and (3.5) are solved numerically with $\sigma(\tilde{T})$ in the form (2.9), in which the quantity t_R varied from 0 to ∞. The critical curves $Q_{0c}(t_R)$ and $k_c(t_R)$ are illustrated in Fig. 4. As $t_R \to \infty$, $Q_{0c}(t_R)$ tends asymptotically to $6.4t_R$, and $k_c(t_R)$ to $1.46 \log t_R + 3.32$, which is in agreement with the calculation of the critical curve in Sec. 2 (in which $|\ln r_{\sigma 0}|^{-1} \ll T_R$). As $t_R \to \infty$, one can construct an asymptotic expansion of the eigenfunctions and eigenvalues of the problem (2.15), (3.4), (2.24), and (3.5) in powers of the small parameter t_R^{-1}, which reduces in the zeroth approximation to the problem (2.15), (2.19), (2.24), and (2.23); however, this will not be done in view of the obvious simplicity of the construction.

The eigenfunctions of the problem (2.15), (3.4), (2.24), and (3.5) outside the electrically conducting region are plotted in Fig. 5 with $\sigma = \sigma(\tilde{T})$ in the form (2.26) as $t_R \to 0$ and for k = 4 (the eigenfunctions in the electrically conducting zone will look the same as in Fig. 2). It is evident that the appearance of these curves ($t_R \to 0$) is very similar to the appearance of the eigenfunctions illustrated in Fig. 3 ($t_R \to \infty$), although it is not necessary, of course, to forget about the coefficients which appear in front of the eigenfunctions in the asymptotic expansions (2.13), (2.18) and (3.2), (3.3), as well as the fact that the vector of the eigenfunctions is determined to an accuracy of a constant (not dependent on r) factor.

The critical curves Q_{0c} and k_c for the problem (2.15), (3.4), (2.24), and (3.5) were constructed numerically with variation of the electrical conductivity distribution as $t_R \to 0$. The electrical conductivity $\sigma = \sigma(\tilde{T})$ was specified with the following single-parameter family of curves:

$$\sigma(\tilde{T}) = \begin{cases} \dfrac{e^{\alpha \tilde{T}} - 1}{e^{\alpha} - 1} & \text{for} \quad \tilde{T} > 0, \\ 0 & \text{for} \quad \tilde{T} < 0 \; (-\infty < \alpha < \infty). \end{cases}$$

It is evident that the distribution $\sigma(\tilde{T})$ is fullest for large negative α, and on the contrary, it is least full as $\alpha \to \infty$; the electrical conductivity distribution becomes a linear function of the temperature as $\alpha \to 0$.

The calculated critical curves $Q_{0c}(\alpha)$ and $k_c(\alpha)$ are shown in Fig. 6. It is evident that the filled distribution $\sigma(\tilde{T})$ ($\alpha \to -\infty$) is more stable than the distribution $\sigma(\tilde{T})$ as $\alpha \to \infty$. As $\alpha \to -\infty$, $Q_{0c}(\alpha) \to 18$, and as $\alpha \to \infty$, $Q_{0c}(\alpha) \to 0$. However, the dependence of Q_{0c} on α is nonmonotonic, as follows from Fig. 6. Upon the variation of α from ∞ to -6 the quantity Q_{0c} increases from 0 to 18.8, and upon a further decrease of α from -6 to $-\infty$, Q_{0c} decreases from 18.8 to 18.

One should note that the critical value of the wave number k_c does not depend on α. This is easy to show. Actually, one can extract a closed system of sixth-order equations from the system (2.15) to describe the electrically conducting region [these are the first five equations in the system (2.15)], whose order can be lowered from seventh to sixth by the introduction of the new function

$$\tilde{u}_1(\zeta) = d\tilde{w}_1/d\zeta, \tag{3.6}$$

with the five boundary conditions: three conditions of boundedness

$$\tilde{u}_1 = d\tilde{\theta}_0/d\zeta = \tilde{h}_0 = 0 \quad \text{at} \quad \zeta = 0 \tag{3.7}$$

and the two conditions

$$\tilde{p}_0 = \tilde{h}_0 = 0 \quad \text{at} \quad \zeta = \gamma;$$

i.e., the functions $\tilde{u}_1$ and $d\tilde{\theta}_0/d\zeta$, which are necessary for splicing the exterior and interior asymptotic expansions, are determined to an accuracy of a constant factor, e.g., A_1.

There is also a system of sixth-order equations [this is the system [2.4)] for the description in the zeroth approximation of the region where $\sigma = 0$ with the five boundary conditions: three boundary conditions

$$v_0 = w_0 = \theta_0 = 0 \quad \text{at} \quad r = 1$$

and the two conditions $v_0 = \theta_0' = 0$ at $r = 0$; i.e., the functions $w_0'(r)$ and $\theta_0(r)$ necessary for the splicing are determined to an accuracy of a constant factor, e.g., A_2.

Keeping (3.6) in mind, we obtain from the splicing conditions (3.5)

$$A_1 k^2 Q_0 \gamma \tilde{u}_1(\gamma) = A_2 \lim_{r \to 0} (r w_0'),$$

$$A_1 \gamma d\tilde{\theta}_0/d\zeta|_{\zeta=\gamma} = A_2 \theta_0|_{r \to 0},$$

whence for $A_1 \neq 0$ and $A_2 \neq 0$ we have

$$Q_0 = \left[(d\tilde{\theta}_0/d\zeta)_{\zeta=\gamma} / \tilde{u}_1(\gamma)\right]\left[\left(\lim_{r \to 0} (r w_0')\right)/(k^2 \theta_0(0))\right]. \tag{3.8}$$

It should be noted that $\tilde{u}_1(\gamma)$ and $d\tilde{\theta}_0/d\zeta|_{\zeta=\gamma}$ are functions only of α [this follows from the form of Eqs. (2.15) and the boundary conditions (3.7)], and $\lim_{r \to 0} (r w_0')$ and $\theta_0(0)$ are functions of k and t_R [this is evident from (2.4) and (2.9)]. Then one can write Eq. (3.8) for Q_0 in the form

$$Q_0 = f_1(\alpha) f_2(k, t_R).$$

The critical value $k = k_c$ is determined by the condition

$$dQ_5/dk = f_1(\alpha)df_2(k,\ t_R)/dk = 0$$

or

$$(d/dk)f_2(k,\ t_R) = 0,$$

from which it is evident that the critical value of the wave number k_c does not depend on α, or more generally, on the distribution $\sigma(\tilde{T})$.

In conclusion of this investigation of the stability of a thin electric arc to symmetrical perturbations one should note the following:

1) The stability boundary for as thin an electric arc as desired (the remaining parameters are fixed) is determined by the value of the quantity $Q\beta^2\varepsilon^2\varepsilon_1^2/T_R \ln \varepsilon_1$; the critical value of this parameter is equal to −6.4, and the critical value of the wave number tends to infinity;

2) in the case in which the wall temperature is comparable to or less than $|\ln r_{\sigma 0}|^{-1}$, the stability boundary is determined by the value of the quantity $Q\varepsilon^2\varepsilon_1^2$; depending on the wall temperature, the critical value of this quantity varies from 14 to ∞, and the critical wave number varies from −3.01 to ∞; and

3) upon variation of the electrical conductivity distribution as a function of the temperature it turns out that the fullest distribution is the most stable (although more accurately, this dependence is nonmonotonic), and the critical wavenumber does not depend on the electrical conductivity distribution.

All the calculations were performed on a computer, and the method of linearly independent solutions (for example, see [4]) was used in the numerical calculations; the relative accuracy of the calculated curves is no less than 10^{-2}.

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